How many unmanned air vehicles can fly in a given airspace?

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Abstract: The paper examines the technicalities of multiple unmanned air vehicles (UAV) operations from the viewpoint of the underlying flight dynamics and physical constraints of moving in a defined airspace. Simulation results are presented and possible physical laws relating the probability of collision with the UAV density, speed and size will be proposed. Implications for design of UAV air regulations with reference to CAAS guidelines and FAR part 107 will be discussed.

1. Introduction

One recent development in air transportation is the proposed use of unmanned air vehicles (UAV) as air taxis and last-mile delivery service. Sub-scale prototypes of such UAVs are already being tested by companies like A³ (Vahan), Eviation (Orca).

This will lead to distributed air traffic management where each UAV has its own objective and has to detect and avoid other UAVs in the same airspace. The situation is similar to a flock of birds or bats flying in the same airspace. One basic consideration of distributed management is the maximum number of UAVs that can safely operate in a given airspace. In this study we will examine this problem from the underlying physics of multi-body dynamics and stochastic processes. In section 2, the mathematical model will be defined. Section 3 presents the simulation and statistical results. Recommendations for future air regulations will be discussed in section 4.

2. Mathematical Model

In order to identify the dominant factors affecting multiple UAV traffic management, we define the flight dynamics of each UAV in a horizontal layer of airspace by

\[
\begin{align*}
\frac{dX}{dt} &= V \cos \psi \\
\frac{dY}{dt} &= V \sin \psi
\end{align*}
\]
where X, Y are the Cartesian coordinates of the UAV and $\psi$ is the heading of the UAV measured from the X axis. Each UAV has a circular size of radius r and all the UAVs operate in a circular flight area of radius R with speed V. This is illustrated in fig 2.1.

For better numerical conditioning we non-dimensionalize equations (2.1)-(2.2) as follows:

Let the coordinates be non-dimensionalized by the radius R

$$x = X/R \quad y = Y/R \quad (2.3)$$

and let the non-dimensional time be

$$\tau = (V/R) t \quad (2.4)$$

The non-dimensional kinematic equations are:

$$dx/d\tau = \cos\psi \quad (2.5)$$
$$dx/d\tau = \sin\psi \quad (2.6)$$

This shows that for the simulation we may set the flight area radius R to 1 with the non-dimensional size $\rho$ of the UAV specified as a fraction of R, $\rho = r/R$. The speed V of the UAV need not be specified as it serves merely to rescale time.

In addition to the flight dynamics we provide each UAV with a simple sense and avoid mechanism. Should any two UAVs be involved in a close encounter i.e. they come within a distance of $2\rho$ from each other, each UAV will turn to its right by a random amount between 0 and 90°. We also need to maintain the same number of UAVs in the flight area throughout the simulation. This is ensured by making each UAV turn back in a random direction should it exceed the boundary of the flight area.

3. Simulation Results

Each simulation run starts with the UAVs placed at random on the boundary with a random heading $\psi$ moving into the flight area. We vary the number of UAVs N from 10 to 100 in steps of 10 and the UAV size $\rho$ from 0.01 to 0.1 in steps of 0.01. Each simulation run uses a time step size $d\tau = 0.01$ with a run time of 1000 non-dimensional time units.

For each run two quantities of interest for air traffic management are

![Fig 2.1 Flight area with UAVs in close encounter (red circle)](image)

![Fig. 3.1 Estimated $E[p_c]$ as a function of number of time steps, $N = 10$, $\rho = 0.1$, $d\tau = 0.01)](image)
estimated using the Robbins-Monro stochastic approximation method [1]:

1) $E[p_c]$: The expected fraction of UAV in close encounter.
2) $E[\tau_c]$: The expected non-dimensional time between close encounters.

The expected dimensional time between close encounters is then $E[T_c] = R/V E[\tau_c]$. A typical plot of the estimated $E[p_c]$ for one run is shown in fig. 3.1 as a function of the number of time steps. We note that the simulation time of 1000 non-dimensional time units is sufficient to ensure convergence of the estimate.

The results for $E[p_c]$ is shown in figure 3.2 where we plot $\ln(1 - E[p_c])$ as a function of the UAV density $N(r/R)^2$. The strong linear dependence indicates that the expected fraction of UAVs in close encounter follows an approximation of the form:

$$E[p_c] = 1 - \exp[-k N (r/R)^2] \quad (3.1)$$

where $k \approx 3.56$ from a least-squares fit of the data. This approximation holds well for the low to moderate UAV density regime, $N(r/R)^2 < 0.5$, which is of prime interest for air traffic management.

Figure 3.3 shows a typical distribution of the time between near encounters $\tau_c$. The mean and standard deviation were close, $\mu = 0.80$ and $\sigma = 0.82$ and this suggests that $\tau_c$ follows an exponential distribution:

$$P(\tau_c > t) = 1 - \exp[-\tau_c / \mu] \quad (3.2)$$

where $E[\tau_c] = \mu$. Similar behaviour was observed for all the other runs.

The variation of $\mu$ is shown in Fig. 3.4 as a function of the quantity $VN r/R$. The data suggests an approximation for $E[T_c]$ of the form:

$$E[T_c]) = \frac{0.43}{(VN 2r) / (\pi R^2)} \quad (3.3)$$

The quantity $VN 2r$ is the “rate of swept area” used in the calculation of the mean-free path in gas kinetic theory. This quantity featured in early attempts to model air traffic
using a gas kinetics model [2-3].

4. Recommendations for future air regulations

Current UAV air regulations [4-5] require that UAVs must be flown within visual line of sight (VLOS) of the operator. For future UAVs with sense and avoid capabilities, these regulations will not be applicable. Presently we will show how quantities like \( E[p_c] \) and \( E[T_c] \) may be used to draft future UAV air regulations:

Given the radius of the airspace \( R \), size \( r \) and speed \( V \) of the UAVs

1. Set the maximum acceptable fraction of UAVs in close encounter \( p \).
2. Solve \( E[p_c] < p \), using relation (3.1)
   \[
   1 - \text{Exp}[-3.56 \frac{N}{(r/R)^2}] < p \quad (4.1)
   \]
3. Set the minimum acceptable time between close encounters = \( T \)
4. Solve \( E[T_c] > T \), using relation (3.3)
   \[
   0.215 \frac{(\pi R^2)}{(V N r)} > T \quad (4.2)
   \]
5. Pick the smaller bound for \( N \) from the solution of the two inequalities (4.1) & (4.2). This provides a guide for determining how many UAVs can fly in a defined airspace.

For a quantitative example, Consider an airspace of \( R = 1 \) km with UAVs of size \( r = 1 \) m flying at 10 m/s. We select a safety level of \( p = 0.001 \) and \( T = 200 \) s.

Inequality (4.1) \( \Rightarrow n \leq 281 \) (refer fig 4.1)
Inequality (4.2) \( \Rightarrow n \leq 337 \) (refer fig 4.2)

Hence the maximum number of UAVs that can operate in the airspace is \( N = 281 \). Note that other considerations e.g. factor of safety and communication bandwidth requirements may reduce this limit further.
References


